April 24, 1890.

Sir G. GABRIEL STOKES, Bart., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read :—

I. "On a Pneumatic Analogue of the Wheatstone Bridge." By W. N. SHAW, M.A., Lecturer in Physics in the University of Cambridge. Communicated by LORD RAYLEIGH, Sec. R.S. Received March 31, 1890.

When fluid flows steadily through an orifice in a thin plate, the relation between the rate of flow, V, measured in units of volume of fluid per second, and the head H (the work done on unit mass of the fluid during its passage) may be expressed by the equation :—

$$H = RV^2,$$

where R is a constant depending upon the area of the orifice, If the head be measured in gravitation units, R is equal to $1/2gk^2a^2$, where g is the acceleration of gravity, a the area of the orifice, and k the coefficient of contraction of the vein of fluid, a factor which is independent of the rate of flow.

Let us suppose a current of *incompressible* fluid to be drawn in succession through two orifices, a_1, a_2, arranged one at each end of a closed space, B, so large that there is no appreciable difference of head between different parts of it and that the kinetic energy of the flow through the one orifice does not affect the flow through the other. By the principle of continuity, the flow V will be the same through each of the orifices, and we have for the head H_1 between the two sides of the orifice of entry, $H_1 = R_1V^2$, and for the head H_2 between the two sides of the orifice of exit, $H_2 = R_2V^2$, where R_1 and R_2 are corresponding constants for the two orifices. From the definition of the term "head," it follows that $H_1+H_2(=\mathfrak{h})$ is the total head between the outside of the second orifice and the outside of the first. We may therefore regard H_1 and H_2 as partial heads which make up the total head $\mathfrak{h}$. We may suppose that the head $\mathfrak{h}$ is due to a constant manometric depression maintained in a second large closed space, A, communicating with the first space, B_1, by means of

the second orifice. If now we have a third closed space, B_2, likewise provided with two orifices, a_3, a_4, one of which, a_4, communicates with the space A where the constant manometric depression is maintained, while the other, a_3, is open to the same supply of fluid as that which feeds a_1, we get a second flow, V′, which we may speak of as being in multiple arc with the first, and to which the following equations apply:—

$$H_3 = R_3 V'^2,$$

$$H_4 = R_4 V'^2,$$

$$H_3 + H_4 = \mathfrak{h}.$$

H_3 and H_4 are the partial heads for the second flow, and R_3, R_4 the constants for the orifices a_3, a_4 respectively.

We have, therefore, an arrangement for the flow of fluid analogous to the arrangement of the Wheatstone quadrilateral for the flow of electricity, the galvanometer circuit being supposed open. The head $\mathfrak{h}$ corresponds to the electromotive force of the battery, V^2 and V'^2, correspond to the electric currents in the two branches; R_1, R_2, R_3, R_4, to the four electrical resistances; the spaces A, B_1, B_2, take the places of the brass connecting blocks of a Post Office box or the copper connexion pieces of a metre bridge. The partial heads, H_1, H_2, H_3, H_4, correspond to the electromotive forces between the ends of the four several wires. Making contact with a key in the galvanometer circuit would correspond to opening a tube of communication between the spaces B_1, B_2, above mentioned, and the hydrodynamic condition corresponding to no current through the galvanometer would evidently be the condition of no flow of fluid through the tube, and the galvanometer must be represented by some apparatus for detecting a flow of fluid; the detector need not, however, be designed to measure a flow any more than the galvanometer need be suitable for measuring a current. The condition for no flow in the "galvanometer" tube is that there should be no head between its ends; this condition is satisfied if $H_1 = H_3$ or $H_2 = H_4$; from which it follows that the condition is entirely independent of the total head, $\mathfrak{h}$, and depends only on the constants of the four orifices; we have, in fact, the ordinary Wheatstone-bridge relation:—

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}.$$

If the coefficients of contraction may be assumed to be independent of the shape of the orifice, we get the condition for no flow through the "galvanometer" tube:—

$$\frac{a_1}{a_2} = \frac{a_3}{a_4},$$

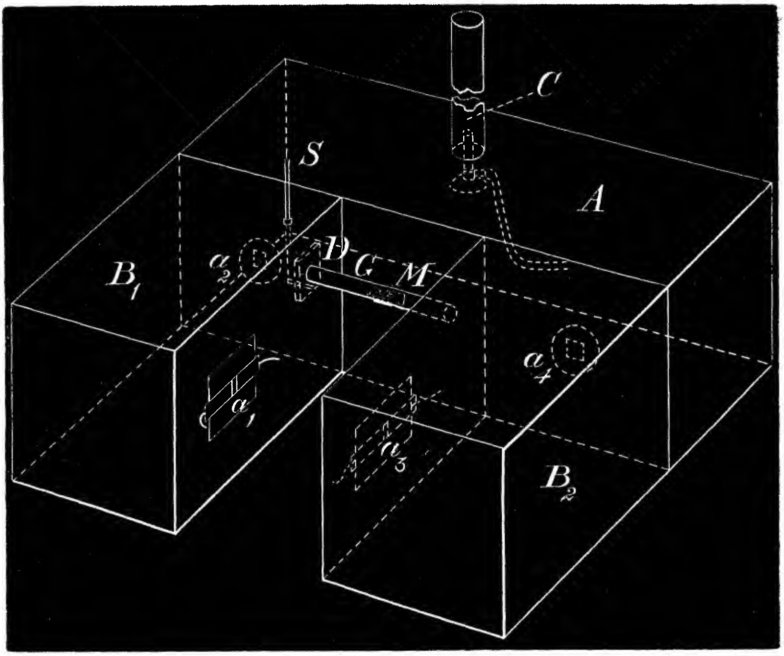
where the a's represent the actual measured areas of the four orifices.

It is evident that the practical realisation of this hydrodynamic analogue is by no means difficult. We require simply a "source" and a "sink" communicating, the one with the other, by two pairs of apertures in two separate boxes, which must be of such a size that the kinetic energy of the entering streams is practically completely dissipated in the boxes. The two boxes must also be connected by a tube in which there must be placed an apparatus for detecting the existence of a flow between the boxes.

The hydrodynamic analogue suggested itself to me in the course of the study of a number of problems in ventilation depending upon the flow of air between nearly-closed connected spaces, for example, adjoining rooms. In such cases the differences of pressure which produce the flow are very minute, amounting perhaps to a few hundredths of an inch of water, and the corresponding variations in the density of air may be safely disregarded. Under such circumstances the air will follow the laws of flow of an incompressible fluid, and equations identical with those quoted above will hold for the flow of air.

Measurements made upon the flow of air in order to determine the coefficient of contraction have been hitherto such as may be termed "absolute"; that is to say, the head and the flow have each been separately expressed in absolute measure and the value of R determined by taking the ratio of the head to the square of the flow. This process is exactly analogous to measuring the electrical resistance of a wire by finding the electromotive force between its ends and the current which flows along it.

M. Murgue, in a work on 'The Theory and Practice of Centrifugal Ventilating Machines' (translated by A. L. Steavenson), has shown that the internal resistance of a centrifugal fan to the flow of air through it can be calculated from the effects produced on the flow by varying the size of a second orifice through which the air has to pass. This process is evidently parallel to calculating the internal resistance of a battery by finding the effect produced upon the current by varying the external resistance. The development of the electrical analogy seems to afford a novel method of comparing resistances to the motion of air, and of verifying the laws of flow, and one which requires only a detector and not an anemometer, and is independent of the constancy of the flow. Whether it could be used practically to test the laws of flow and measure the pneumatic constants for various orifices to a higher degree of accuracy than has hitherto been attained, evidently depends upon the sensitiveness of the arrangement. In order to try this, I have had constructed what may be called a pneumatic analogue of the Wheatstone Bridge. It is represented in fig. 1, and consists of three wooden boxes, A, B_1, B_2.



A is 4 ft. × $1\frac{1}{2}$ ft. × $1\frac{1}{2}$ ft., and B_1 and B_2 are each 3 ft. × $1\frac{1}{2}$ ft. × $1\frac{1}{2}$ ft. The ends of B_1 and B_2 abut against the side of A, as shown in the figure; between B_1 and A is a rectangular opening, a_2, 1 in. × $\frac{1}{2}$ in., in a cardboard diaphragm, and between B_2 and A a rectangular opening, a_4, 1 in. × 1 in., in a similar diaphragm. In the side of B_1 at a_1 is an adjustable slit, made by cardboard shutters sliding in cardboard grooves, and at a_3 in the side of B_2, opposite to a_1, is a similar adjustable slit. The tube connecting B_1 and B_2, or "galvanometer" tube, is a straight tube of glass, G, of about 1·1 inch internal diameter. It can be closed at one end by a small trap-door, D, in the interior of the box B_1, which can be opened and shut by a steel wire, S, passing through a cork in the top of B_1. The sensitiveness of the apparatus depends upon the indicator employed. There are many indicators that might be employed; the one I have tried and have found to work well consists of two very small parallel sewing needles, stuck through a cap of elder-pith, supported on a small agate compass centre; the needles carry very light mica vanes on one side of the centre, counterpoised by a small quantity of platinum wire. The whole is balanced on the point of the finest needle I could obtain, and forms a very delicate wind vane. When first mounted, the needles always took up a position of equilibrium with the points

northward, although they had not been intentionally magnetised, nor, indeed, exposed to any risk of their being so from the time of their being purchased. They were, no doubt, very slightly magnetised, but the time of swing was very long, and the position of equilibrium not sufficiently definite. I therefore magnetised them more strongly; the little vane then took up, in consequence, a definite position of equilibrium with the planes of the vanes approximately north and south. The apparatus being so placed that the tube, G, is east and west, the vanes always set across the tube when there is no current. The needle points enable the position of equilibrium to be clearly identified by the aid of a fiducial mark on the glass tube. The sensitiveness can be altered as desired by an external control magnet, just as that of a galvanometer needle can be. The little compass needle or wind vane, M, is very sensitive to the motion of air in the tube, and although it may be possible to find other detectors that are equal, or even superior to it, yet the ease of seeing it, the rapidity of its action, and its definite zero are decidedly in its favour.

The head is produced by a gas burner in a metal chimney, C, fitted to the lid of the box A.

Various precautions are required in fitting the boxes together to secure that the air should only flow through apertures intended for its passage, but they need not here be detailed. They consist mainly in the plentiful application of glue and brown paper.

The apparatus was designed when I was making a number of observations of flow of air to illustrate the theory of ventilation, and I did not anticipate that any high degree of accuracy could be aimed at. I was, therefore, agreeably surprised to find that the identification of the condition of no flow is capable of much greater accuracy than the arrangements for measuring the areas of the orifices would allow me to interpret.

Of the four apertures of the bridge, two, viz., a_2 and a_4, are inaccessible without pulling the arrangement to pieces; they represent areas of $\frac{1}{2}$ sq. in. and 1 sq. in., respectively, as accurately as a knife could cut them in cardboard.

The other two areas, viz., a_1 and a_3, are made by sliding shutters, as already mentioned. Their edges were cut with a knife, and they probably are only rough approximations to areas in a truly thin plate, so that little importance can be attached to the final results of the measurements which will be given below; they serve only to show that the width of the adjustable slit, when there is no flow through the galvanometer tube, is a perfectly definite magnitude.

The following observations have been taken with the apparatus:—

I. *To Verify the Law of Proportionality of Areas, viz.,* $\frac{a_1}{a_2} = \frac{a_3}{a_4}$.

As already stated, $a_3 = 0·5$ sq. in., $a_4 = 1$ sq. in., so that if the law holds a_3 should be found to be equal to $2a_1$. In testing the proportionality, a_1 was made successively equal to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 sq. in., by the use of a cardboard wedge, 343 mm. length of which corresponds to 1 in. breadth; and a_3 was adjusted till there was no flow through the galvanometer tube, and the width measured by means of the cardboard wedge.

The measurements were referred directly to an inch scale by parallel-jaw callipers.

The observations are contained in the following table:—

Table I.

Area of a_1 in square inches	·25.	·50.	·75.	1·00.
Observation of width of a_3 in divisions of the wedge	165 165 166 166 165	337·5 (?) 333·5 333·5 333·5 334	495·5 496 495 494·5 494·5	[68·5]* [68·5] [69·0] [68·0] [67·0]
Means.................	165·4	334·4	495·1	[68·2]
Equivalents in square inches	·465	·985	1·46	1·92
$\frac{a_3}{a_1}$.....................	1·86	1·97	1·95	1·92

The differences in the ratios a_3/a_1 for different values of a_1 are considerable, but it must be remembered that the greatest fractional difference, being 1/19th of the whole, would be accounted for by an error of 1/76th of an inch in the adjustment of a_1 to $\frac{1}{4}$ inch, and a cardboard slit, cut with a knife, can hardly be expected to reach beyond that limit of accuracy. It is evident, from the readings given, that the condition of no flow is capable of very accurate experimental definition with the little compass detector.

II. *Verification of the Inference that the Condition of No Flow is Independent of the Total Head.*

This has only been carried so far as to determine whether, when the adjustment of areas was made, the equilibrium could be dis-

* The readings in this column were taken by means of a different and wider wedge.

arranged by altering the quantity of gas burning in the jet. No difference was, however, observed in the position of equilibrium of the needle, whether the gas was quite low, or on full, or turned out, leaving only the head due to the heat of the metal chimney. So far as could be tested in this manner, one of the advantages of the Wheatstone bridge, viz., that the adjustment is independent of the electromotive force, is correctly followed in the pneumatic analogue.*

III. *Comparison of a Circular with a Rectangular Aperture.*

A circular aperture in a brass plate, one-sixteenth of an inch thick, was balanced against a rectangular one, formed by the sliding card-board shutters. The circle was turned to be 1 inch in diameter, and the inner edge of the aperture was bevelled. The observations, when the circle was in the position a_1, were—

$$a_1 = \pi \times (\tfrac{1}{2})^2 = \cdot 785 \text{ sq. in.}$$

Readings for a_3, $\left\{\begin{array}{l} 489\cdot 5 \\ 490 \\ 490 \end{array}\right\}$ Mean, 490.

Whence $a_3 = 1\cdot 446$ sq. in.

$$\frac{a_3}{a_1} = 1\cdot 86. \qquad \frac{a_4}{a_2} = 2.$$

When the circle was in the position a_3,

$$a_3 = \cdot 785 \text{ sq. in.}$$

Readings for a_1, $\left\{\begin{array}{l} 125 \\ 126 \\ 125\cdot 5 \end{array}\right\}$ Mean, 125·5.

Whence $a_1 = \cdot 362$ sq. in.

$$\frac{a_3}{a_1} = 2\cdot 17. \qquad \frac{a_4}{a_2} = 2.$$

The observations were repeated with similar results.

These two values of the ratio a_3/a_1 would be reconciled by assuming that the circular aperture was only equivalent to a rectangle whose area is 0·925 of the circular aperture, and they, therefore, throw doubt upon the idea that circular and square apertures have the same coefficient of contraction, but the rectangular apertures were

* The flow of air through an aperture (a_3) of 1 sq. in. amounts to about 2 cubic feet per minute when the gas is very low, and to 4 cubic feet per minute when it is full on, so that the head can be changed in a ratio of about 4 : 1.

not such close approximations to orifices in truly thin plates as to warrant the acceptance of this result without apparatus of more elaborate construction. Moreover, there is a possibility of slight leak in the grooves of the shutters, which ought not to be disregarded.

The observations are, however, sufficient to show that a properly constructed apparatus is capable of making measurements of the effective areas of orifices with a very considerable degree of precision. It is well known that if the orifice be not an aperture in a thin plate, but in the form of a tube, straight or bent, the flow through the orifice can be represented by an equation of the same form as if the orifice were a thin plate aperture, viz.:—

$$H = RV^2,$$

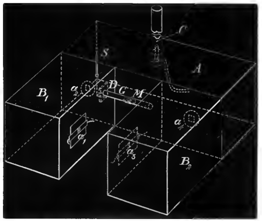
but in the case of a more complicated orifice R cannot be so easily calculated from the dimensions; the value of R might, however, be determined experimentally for an orifice of any shape and dimensions by a pneumatic bridge of suitable size, and the result might be expressed, as M. Murgue suggests for the case of mines in the work already referred to, by stating the area of the thin plate orifice to which the given orifice is equivalent. The comparison of calculated values of R with observed values obtained by a pneumatic bridge would enable us to determine a number of pneumatic constants that are at present only comparatively roughly ascertained, such, for instance, as the coefficient of air friction in tubes of different diameters, the constants of different forms of orifice, the effect of bends and elbows in pipes, and of gauze or gratings covering an orifice. And it would not, I think, be difficult to arrange the apparatus in such a way as to determine the law of resistance of a disc to the passage of air and its variation with velocity. The velocity can be increased to any extent that may be necessary by using a centrifugal fan to produce the head instead of the gas burner.

I am intending, if possible, to have my present apparatus altered in some of its details, so that the orifices may be more definitely expressed in terms of thin plate apertures, and then to use it for the determination of some of the pneumatic constants I have referred to.

II. "On the Effect of Tension upon Magnetic Changes of Length in Wires of Iron, Nickel, and Cobalt." By SHELFORD BIDWELL, M.A., F.R.S. Received April 8, 1890.

Preliminary.

A former communication to the Royal Society ('Roy. Soc. Proc.,' No. 243, 1886, p. 257) contains an account of some experiments relating to the magnetic extensions and contractions of iron wires



C
S
A
B1
D
G
M
a
B2